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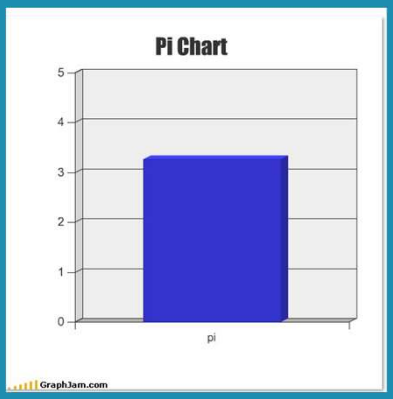
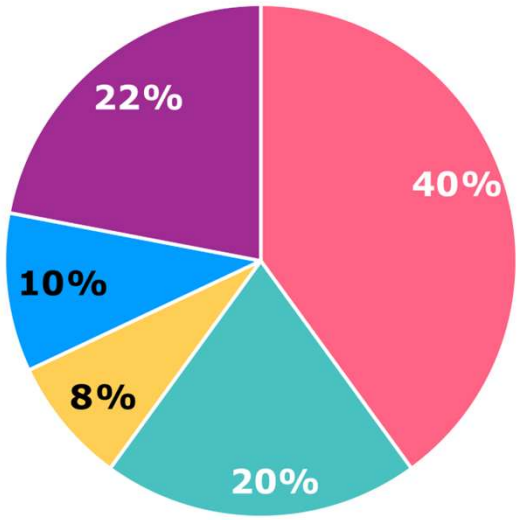
Misconcepties bij statistische grafieken



Prof. Dr. Wim Van Dooren
Centrum voor Instructiepsychologie en –technologie
Faculteit Psychologie en Pedagogische Wetenschappen
Katholieke Universiteit Leuven

1



2 https://commons.wikimedia.org/wiki/File:Misleading_macworld_3d_pie_chart.png KU LEUVEN

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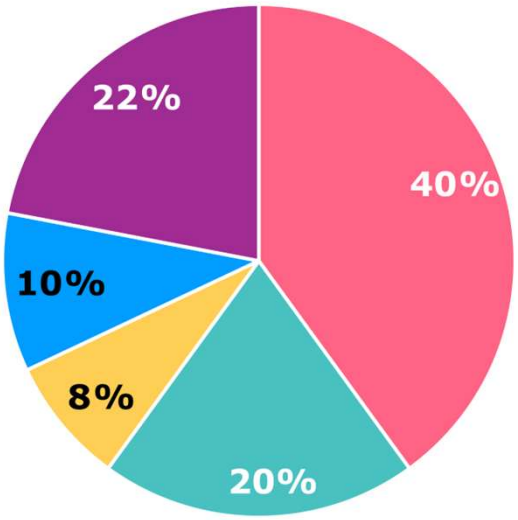

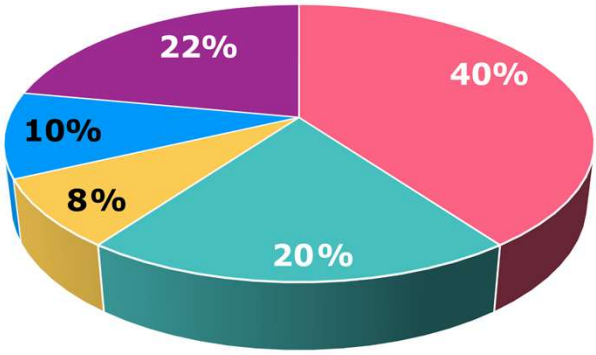

3 https://commons.wikimedia.org/wiki/File:Misleading_macworld_3d_pie_chart.png

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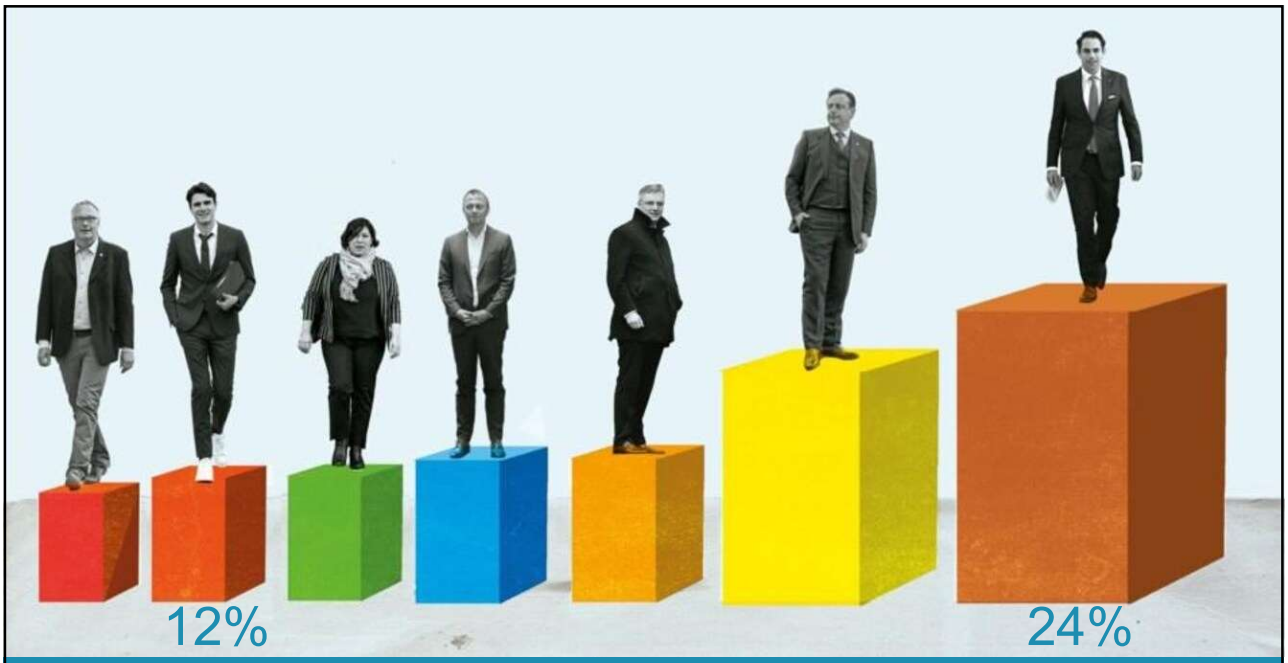

4 De Standaard (23/05/2020)

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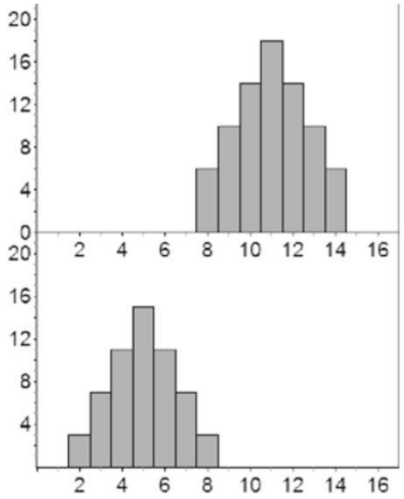
4

Data weergeven in één oogopslag

- GROEP A
- GROEP B



→ *Welke groep heeft gemiddeld de hoogste punten?*

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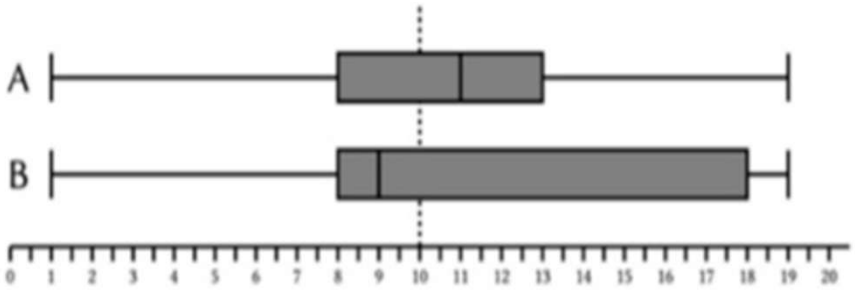
Data weergeven in één oogopslag



→ *In welke groep zijn meer leerlingen geslaagd?*

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Eindterm 06.18 eerste graad

De leerlingen voeren een beschrijvend onderzoek uit met niet-gegroepeerde gegevens van één grootheid

Onderliggende (kennis)elementen

Tabel met absolute frequenties

Staafdiagram, cirkeldiagram, lijndiagram

Mediaan, rekenkundig gemiddelde, modus, variatiebreedte

Memorie

De omvang van de gegevensset moet relevant zijn om een beschrijvend statistisch onderzoek te kunnen uitvoeren. De gegevensset kan al dan niet worden aangereikt. Het voeren van een beschrijvend statistisch onderzoek **veronderstelt een grafische voorstelling, berekeningen en een interpretatie van de resultaten**

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Eindterm 06.18 tweede graad

De leerlingen analyseren statistische gegevens aan de hand van centrum- en spreidingsmaten

Onderliggende (kennis)elementen

- Voorstellingswijzen:
  - Absolute en relatieve frequentietabel
  - Staafdiagram
  - Cirkeldiagram
  - Lijndiagram
  - **Histogram**
  - **Boxplot**
- **Centrum- en spreidingsmaten**
  - Rekenkundig gemiddelde
  - Mediaan
  - Modus
  - Variatiebreedte, interkwartielafstand
  - Standaardafwijking
- Misleidingen

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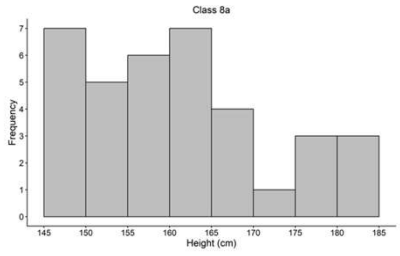
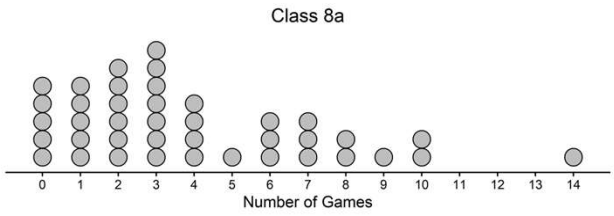
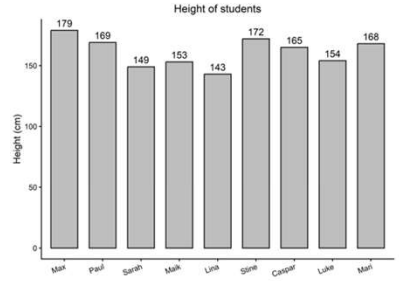
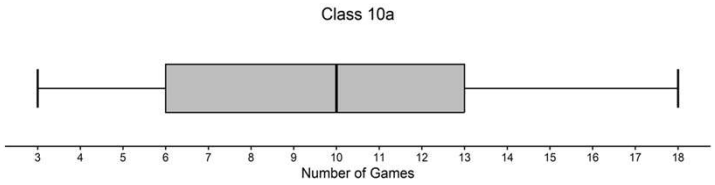
Verdeling van één variabele (centrum, spreiding, scheefheid)









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Onderzoeksperspectief

- Wiskundedidactiek
  - Psychologie van het wiskundeleren en –onderwijzen
- Bouwend op, maar verder uitdiepend dan
  - Algemene didactiek
  - Cognitieve psychologie
  - Wiskunde als discipline

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Onderzoeksperspectief

PME

INTERNATIONAL GROUP FOR THE PSYCHOLOGY OF MATHEMATICS EDUCATION

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Onderzoeksprogramma = teamwork

KU Leuven (sinds 2010)

Stephanie Lem - Wim Van Dooren – Lieven Verschaffel - Patrick Onghena

Freiburg University (sinds 2021)

Frank Reinhold - Martin Abt - Timo Leuders - Katharina Loibl

Universiteit Utrecht (sinds 2018)

Lonneke Boels – Arthur Bakker – Paul Drijvers

Heidelberg University (sinds 2022)

Markus Vogel – Saskia Schreiter – Ayline Heursen

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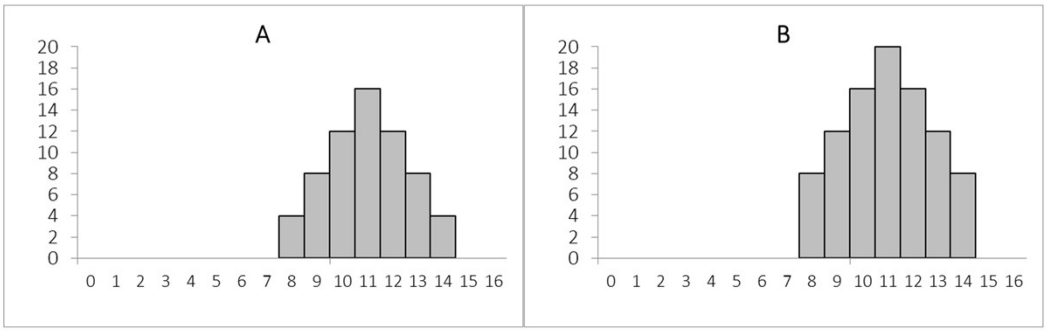

Welke groep haalt gemiddeld de hoogste punten?

a) Groep A
b) Groep B
c) Beide groepen hebben hetzelfde gemiddelde

Lem, S., Onghena, P., Verschaffel, L., Van Dooren, W. (2013). External representations for data distributions: in search of cognitive fit. *Statistics Education Research Journal*, *12* (1), 4-19. doi: 10.52041/serj.v12i1.319

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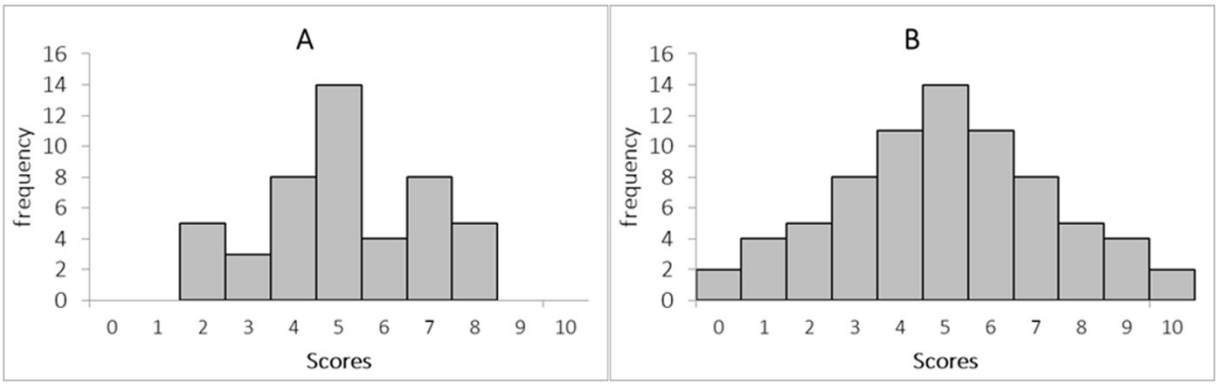

In welke klas hebben de scores het meest spreiding?

a) Klas A
b) Klas B
c) Ze zijn in beide klassen even variabel

Lem, S., Onghena, P., Verschaffel, L., Van Dooren, W. (2013). External representations for data distributions: in search of cognitive fit. *Statistics Education Research Journal*, *12* (1), 4-19. doi: 10.52041/serj.v12i1.319

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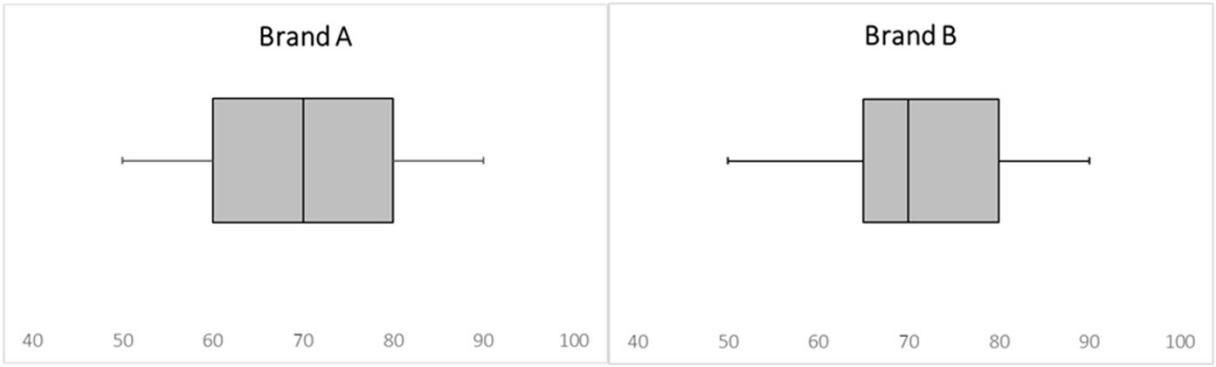

Welk batterijenmerk heeft de grootste spreiding in levensduur?

a) Merk A
b) Merk B
c) Beide merken zijn even variabel

Lem, S., Onghena, P., Verschaffel, L., Van Dooren, W. (2013). External representations for data distributions: in search of cognitive fit. *Statistics Education Research Journal*, *12* (1), 4-19. doi: 10.52041/serj.v12i1.319

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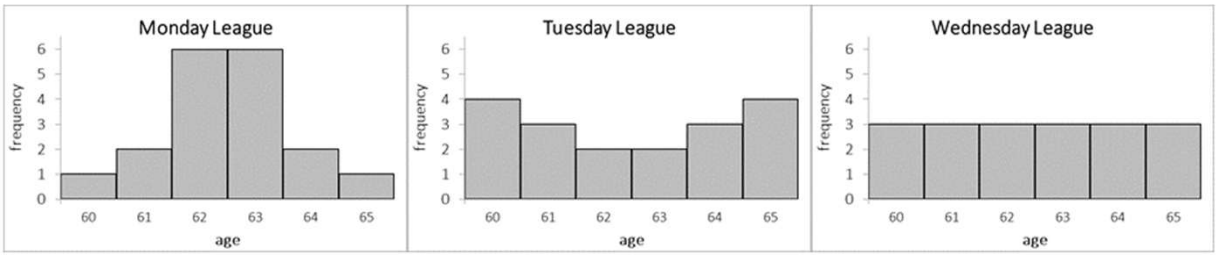

In welke league zit de grootste leeftijdsspreiding?

a) Monday
b) Tuesday
c) Wednesday
d) Alle leagues zijn even variabel

Lem, S., Onghena, P., Verschaffel, L., Van Dooren, W. (2013). External representations for data distributions: in search of cognitive fit. *Statistics Education Research Journal*, *12* (1), 4-19. doi: 10.52041/serj.v12i1.319

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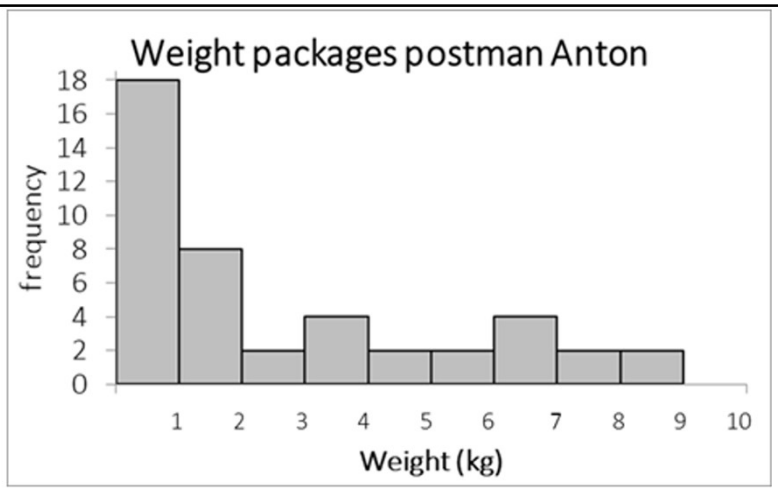

Wat is het gemiddelde gewicht van de pakketjes die postbode Anton rondbrengt?

a) Minder dan 2 kg
b) Tussen 2 en 3,5 kg
b) tussen 3,5 en 4 kg
c) Meer dan 4 kg

Boels, L., Bakker, A., Van Dooren, W., Drijvers, P. (2019). Conceptual difficulties when interpreting histograms: A review. *Educational Research Review*, *28*, Art.No. 100291, 1-23. doi: 10.1016/j.edurev.2019.100291

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MISINTERPRETATIES VAN GRAFIEKEN

- Systematische fouten – geen “vergissingen”
- Vaak ook na onderwijs, en bij experts
- Systematisch onderzoek
  - Experimenteel
    - Via manipuleren van grafiekkenmerken
    - Via reactietijden, oogbewegingen
  - Kwalitatief
    - Via redeneringen van leerlingen
  - Interventie
    - Via onderwijsexperimenten

18

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Wiskunde & Onderwijs 40ste jaargang (2014)

De interpretatie van histogrammen en boxplots – Minder eenvoudig dan op het eerste zicht lijkt

Stephanie Lem, Patrick Onghena, Lieven Verschaffel en Wim Van Dooren

Externe representaties van dataverdelingen, zoals histogrammen en boxplots, zijn alomtegenwoordig. Vooral histogrammen komen we vaak tegen, bijvoorbeeld in kranten en nieuwsprogramma's op televisie. Boxplots worden dan weer vaak gebruikt in technische rapporten en wetenschappelijke publicaties. Toch blijkt uit recent onderzoek dat deze (en andere) externe representaties heel wat interpretatiemoeilijkheden met zich meebrengen.

In dit artikel bespreken we achtereenvolgens histogrammen en boxplots. Voor beide representaties geven we eerst een korte inleiding op de representatie, haar opbouw en geschiedenis. Vervolgens stellen we de belangrijkste misinterpretaties voor. Tot slot bespreken we telkens exemplarisch één of twee onderzoeken naar een specifieke misinterpretatie van de betreffende representatie.

19 Lem, S., Onghena, P., Verschaffel, L., Van Dooren, W. (2014). De interpretatie van histogrammen en boxplots – Minder eenvoudig dan op het eerste zicht lijkt. *Wiskunde en Onderwijs*, *40* (157), 19-32

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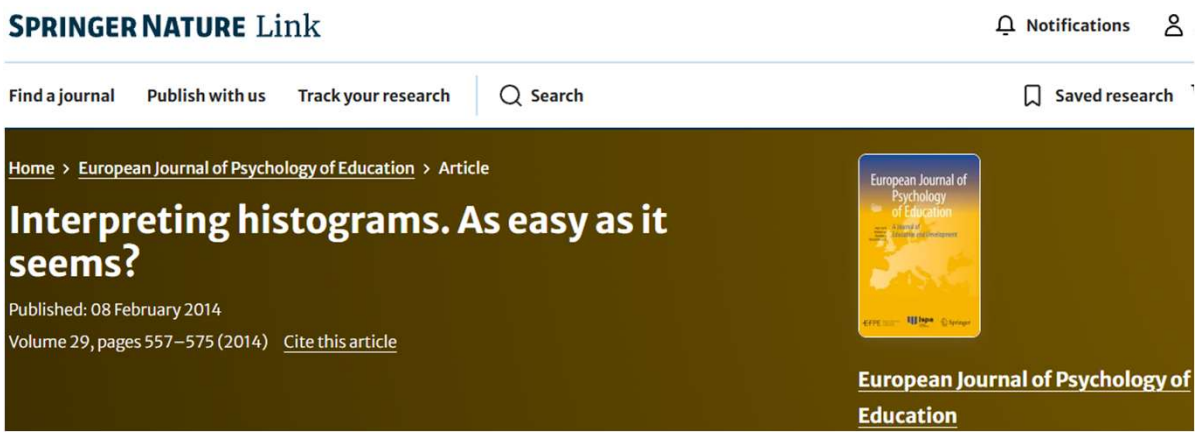

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Interpreting histograms. As easy as it seems?

Published: 08 February 2014

Volume 29, pages 557–575 (2014) Cite this article

European Journal of Psychology of Education

20 Lem, S., Onghena, P., Verschaffel, L., Van Dooren, W. (2014). Interpreting histograms. As easy as it seems? *European Journal of Psychology of Education*, 1-19. doi: 10.1007/s10212-014-0213-x

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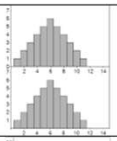
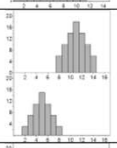
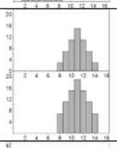
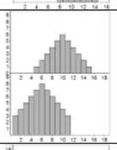
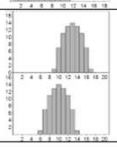
Taakvarianten

	Correct response	Heuristic response	Example
Congruent equal	The same (same location and shape)	The same (same height)	
Congruent unequal	Histogram that is located more to the right	Histogram with highest bars	
Incongruent equal	The same (same location and shape)	Histogram with higher bars	
Incongruent inverse	Histogram located more to the right	Histogram with highest bars	
Incongruent unequal	Histogram located more to the right	The same (same height)	

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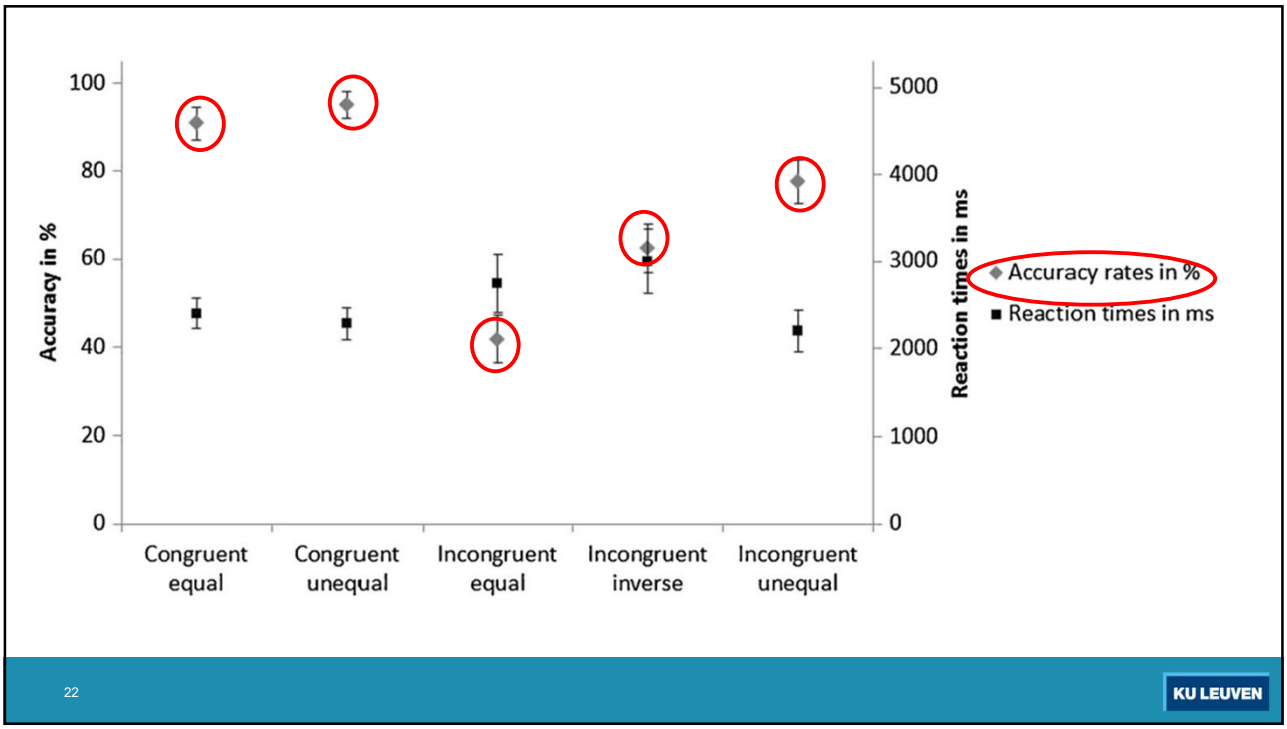

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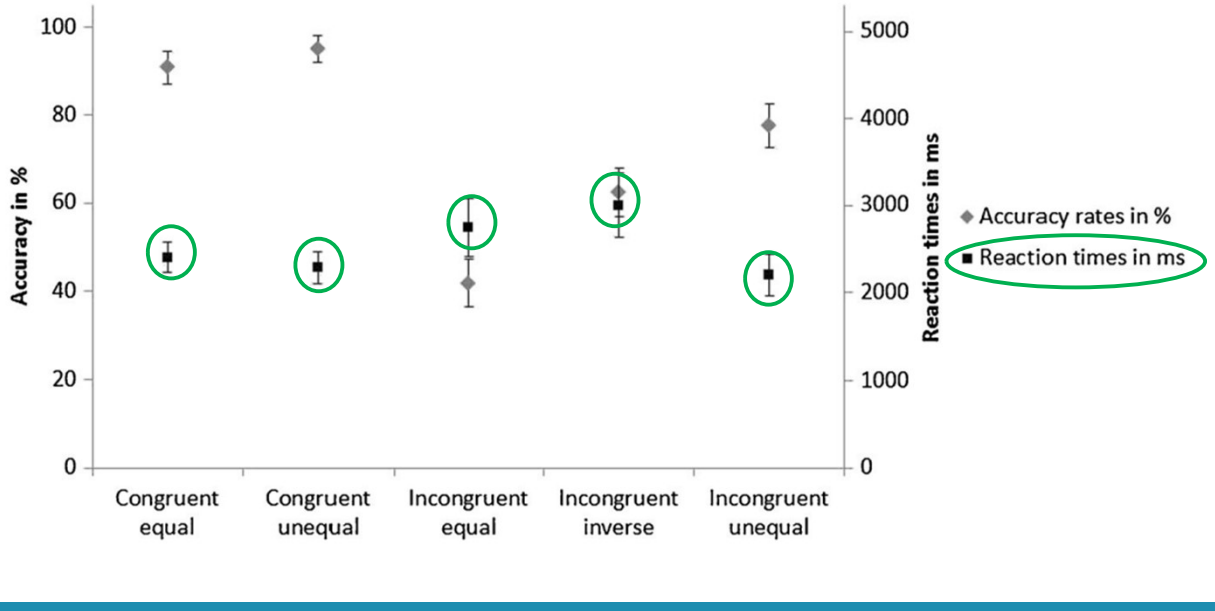

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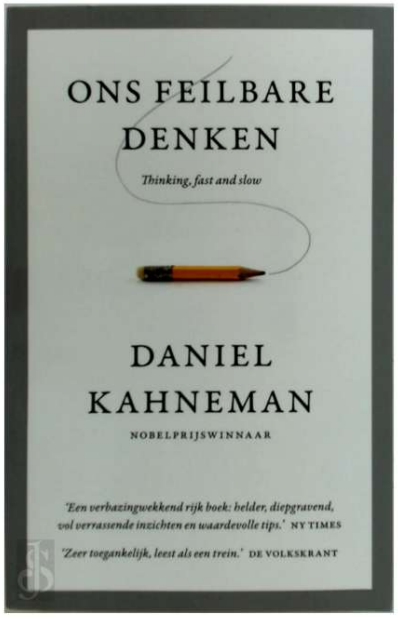

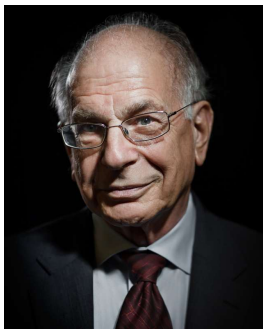

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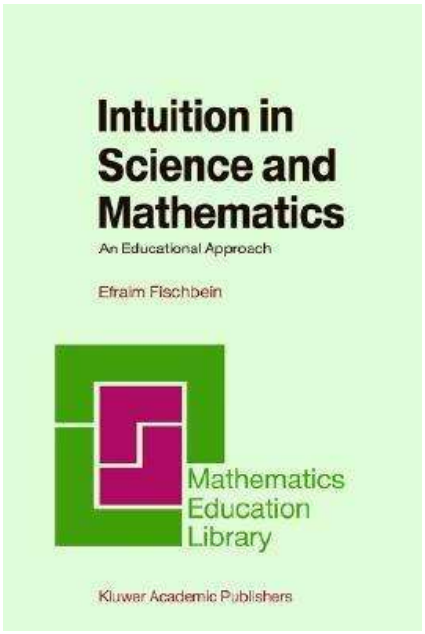

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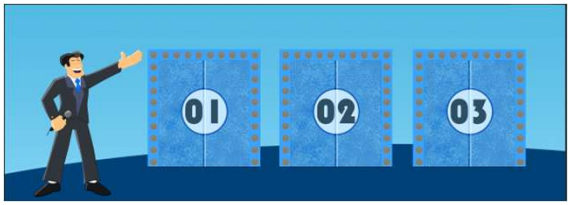
Monty Hall dilemma



Psychologica Belgica

Saenen, L., et al. (2018). Why Humans Fail in Solving the Monty Hall Dilemma: A Systematic Review. *Psychologica Belgica*, 58(1), pp.128-158, DOI: https://doi.org/10.5334/pb.274

THEORETICAL-REVIEW ARTICLE

Why Humans Fail in Solving the Monty Hall Dilemma: A Systematic Review

Lore Saenen*, Mieke Heyvaert†, Wim Van Dooren†, Walter Schaeken* and Patrick Onghena§

The Monty Hall dilemma (MHD) is a difficult brain teaser. We present a systematic review of literature published between January 2000 and February 2018 addressing why humans systematically fail to react optimally to the MHD or fail to understand it.

Based on a sequential analysis of the phases in the MHD, we first review causes in each of these phases that may prohibit humans to react optimally and to fully understand the problem. Next, we address the question whether humans' performance, in terms of choice behaviour and (probability) understanding, can be improved. Finally, we discuss individual differences related to people's suboptimal performance.

This review provides novel insights by means of its holistic approach of the MHD: At each phase, there are reasons to expect that people respond suboptimally. Given that the occurrence of only one cause is sufficient, it is not surprising that suboptimal responses are so widespread and people rarely understand the MHD.

Keywords: Systematic review; Monty Hall dilemma; probability; choice; decision

Saenen, L., Heyvaert, M., Van Dooren, W., Schaeken, W., Onghena, P. (2018). Why Humans Fail in Solving the Monty Hall Dilemma: A Systematic Review. *PSYCHOLOGICA BELGICA*, *58* (1), 128-158. doi: 10.5334/pb.274

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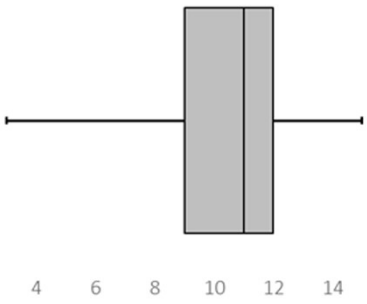
De rol van intuïtie in wiskundig redeneren

- Interpreteren van grafieken (box plots, histogrammen)
- Rationale getallen (interferentie van natuurlijk getalbegrip)
- Reële getallen (interferentie van rationaal getalbegrip)
- Redeneren in niet-proportionele situaties
- Kansrekenen
- …

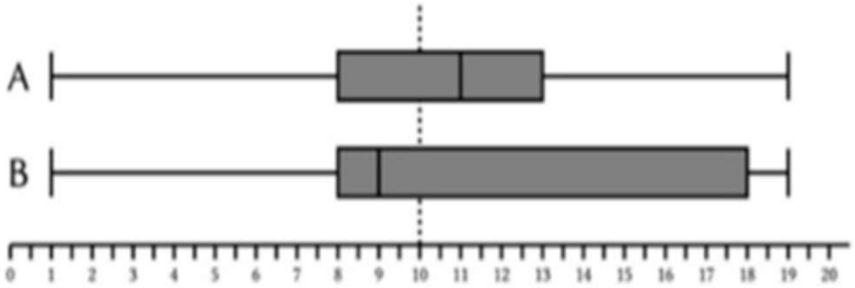
De meest hardnekkige: box plots



Ik heb de lengte van 1000 potloden die op school aanwezig waren gemeten. Welke uitspraak klopt?

a) Er zijn meer potloden tussen 9 en 11 cm dan tussen 11 en 12 cm.
b) Er zijn meer potloden tussen 11 en 12 cm dan tussen 9 en 11 cm.
c) Er zijn evenveel tussen 9 en 11 cm als tussen 11 en 12 cm.

De meest hardnekkige: box plots



→ *In welke groep zijn meer leerlingen geslaagd?*

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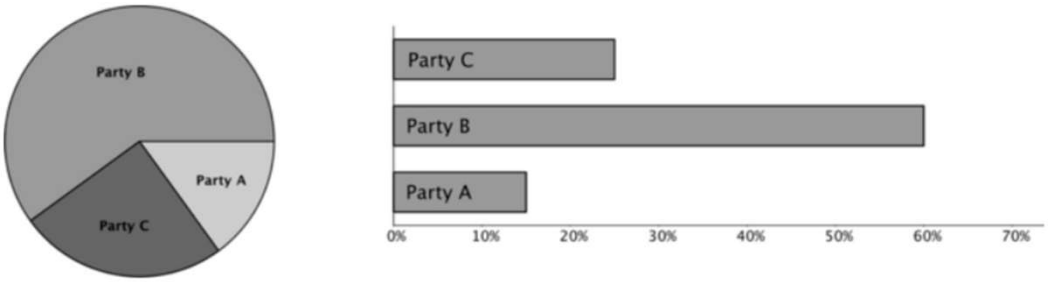
De meest hardnekkige: box plots

"Tegendraadse" manier om oppervlakte te gebruiken

Fig. 2



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De meest hardnekkige: box plots

"Tegendraadse" manier om oppervlakte te gebruiken

Fig. 1

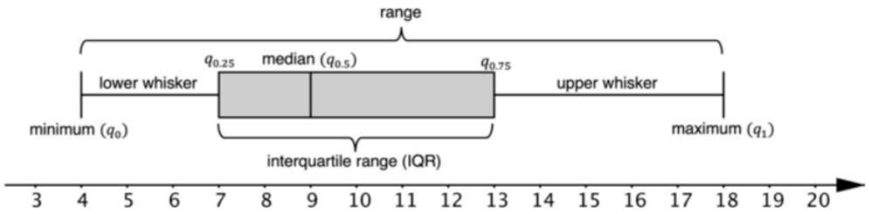
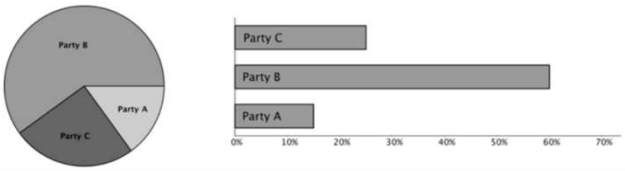


Fig. 2



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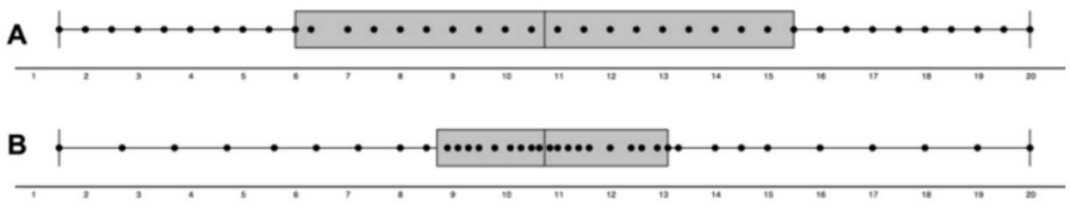
De meest hardnekkige: box plots

"Tegendraadse" manier om oppervlakte te gebruiken

Fig. 3



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Home > Educational Studies in Mathematics > Article

Understanding student errors in comparing data sets with boxplots

Open access | Published: 10 February 2025

Volume 120, pages 169–193 (2025) Cite this article

Educational Studies in Mathematics

Aims and scope →

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Martin Abt, Katharina Loibl, Timo Leuders, Wim Van Dooren & Frank Reinhold

Sections | Figures | References

33 Abt, M., Loibl, K., Leuders, T., Van Dooren, W., Reinhold, F. (2025). Understanding student errors in comparing data sets with boxplots. *EDUCATIONAL STUDIES IN MATHEMATICS*, *120* (1), 169-193. doi: 10.1007/s10649-025-10387-z

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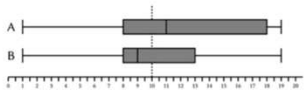
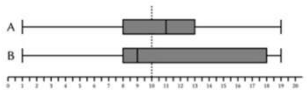
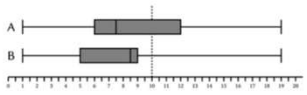
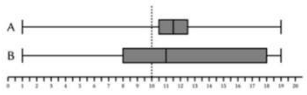
Taakvarianten

	ARFS-congruent	ARFS-incongruent
median items (i.e., the critical value is between both medians—comparing the medians can be considered "correct")	 Applying ARFS leads to the same correct answer (A) as comparing the medians (median schema).	 The correct answer after comparing the medians (median schema) is A, but the use of ARFS leads to the incorrect answer (B).
box items (i.e., one of the two boxes is completely above / below the critical value—comparing the position of the boxes can be considered "correct")	 Applying ARFS leads to the same correct answer (A) as comparing the position of the boxes (box schema).	 The correct answer after comparing the position of the boxes (box schema) is A, but the use of ARFS leads to the incorrect answer (B).

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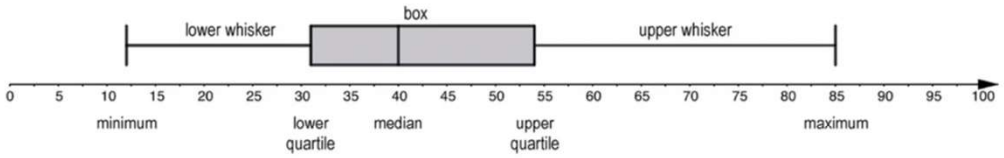
In a class, all students indicated how many minutes they spent on homework the day before.

12 min
16 min
26 min
28 min
30 min
31 min
32 min
34 min
35 min
37 min
38 min
40 min
41 min
43 min
46 min
49 min
51 min
54 min
57 min
65 min
70 min
72 min
85 min



This representation we call ‚***box plot***'.

Click the ‚Next'-Button below to continue with the survey.

1. We sort the list in ascending order of time spent.
2. We determine five characteristic values.
3. We plot the characteristic values over an x-axis.
4. We draw a box and two whiskers.

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item type	ARFS congruency	estimated M	SE	95%CI
median item	congruent	0.963	0.013	[0.927, 0.982]
	incongruent	0.690	0.055	[0.574, 0.786]
box item	congruent	0.948	0.018	[0.901, 0.973]
	incongruent	0.776	0.049	[0.666, 0.857]

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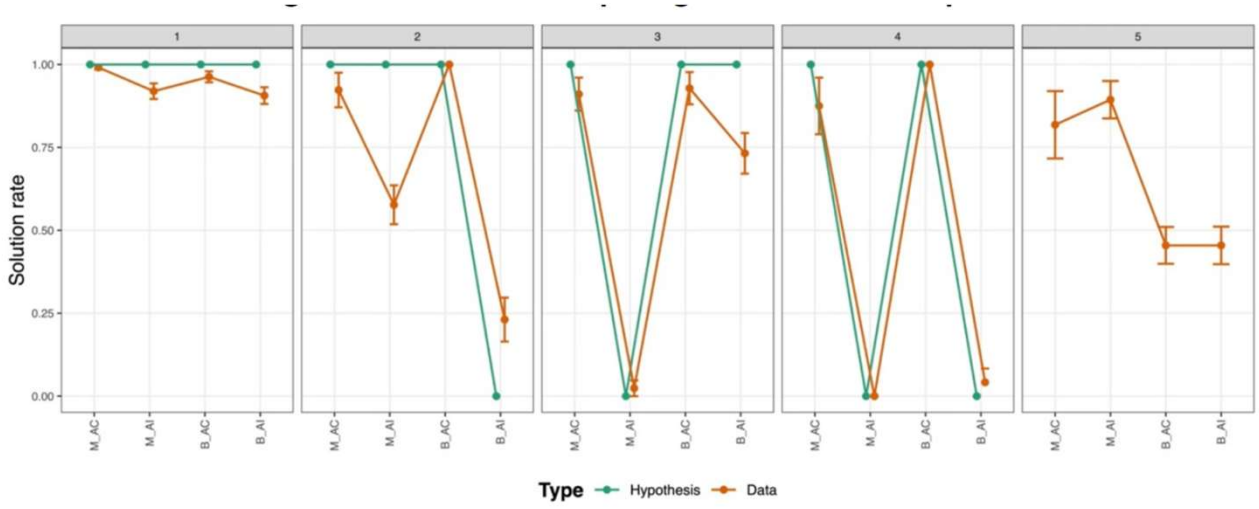

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Statistici, onderzoekers, studenten master in statistics

Psychologica Belgica

Lem, S. et al (2014). Experts' Misinterpretation of Box Plots – a Dual Processing Approach. *Psychologica Belgica*, 54(4), 395-405, DOI: http://dx.doi.org/10.5334/pb.az

RESEARCH ARTICLE

Experts' Misinterpretation of Box Plots – a Dual Processing Approach

Stephanie Lem*, Patrick Onghena†, Lieven Verschaffel* and Wim Van Dooren*

 Lem, S., Onghena, P., Verschaffel, L., Van Dooren, W. (2014). Experts' misinterpretation of box plots - a dual processing approach. *Psychologica Belgica*, *54* (4), 395-405. doi: 10.5334/pb.az

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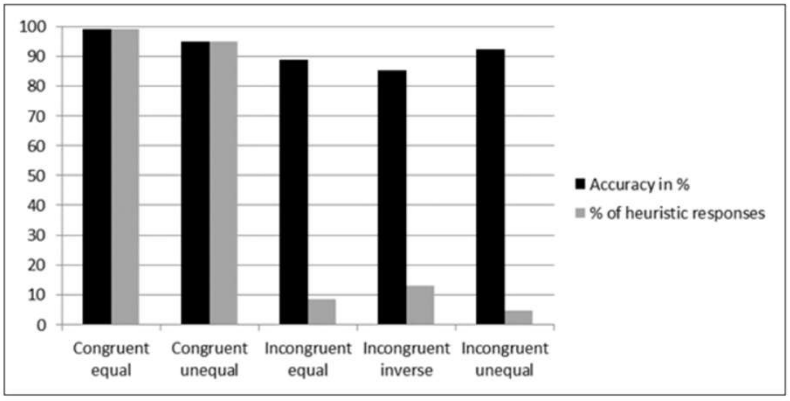

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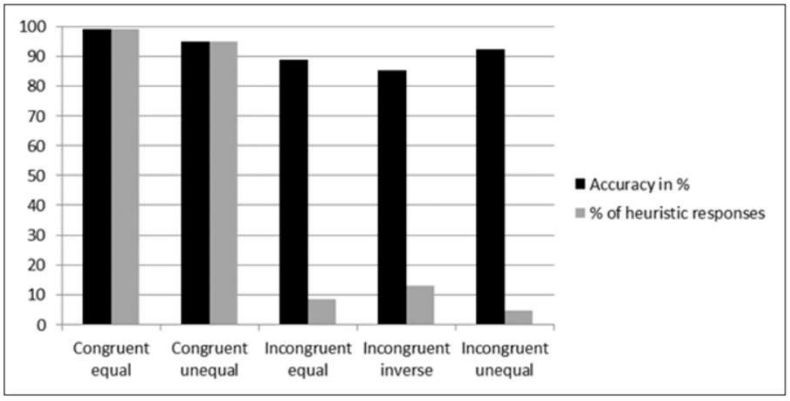

Reactietijden $F(1, 1290) = 4.23$, $p = .040$, Cohen's $d = 0.10$

- Correct op congruente opgaven: 3734 ms, SD = 3534
- Correct op incongruente opgaven: 4179 ms, SD = 4892

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Home › International Journal of Science and Mathematics Education › Article

Combining Multiple External Representations and Refutational Text: An Intervention on Learning to Interpret Box Plots

Published: 19 December 2014

Volume 13, pages 909–926 (2015) Cite this article

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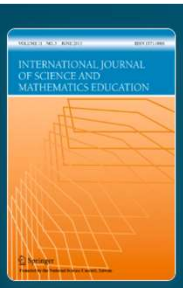

International Journal of Science and Mathematics Education

Aims and scope →

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Stephanie Lem, Goya Kempen, Eva Ceulemans, Patrick Onghena, Lieven Verschaffel & Wim Van Dooren

Sections Figures References

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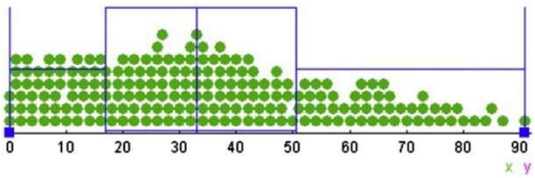

Figure 2. Box plot built on top of a dot plot. Source: Cobb, Gravemeijer, Bowers, and Doorman (1997).

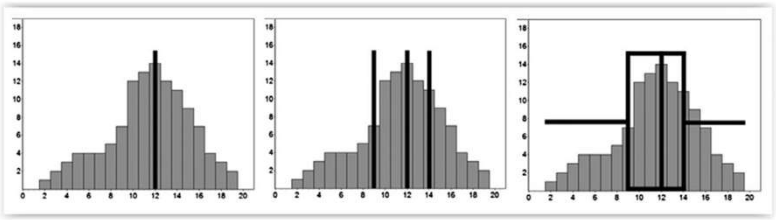

Figure 3. Example of how a box plot was built on top of a histogram in the MERs and combination conditions.

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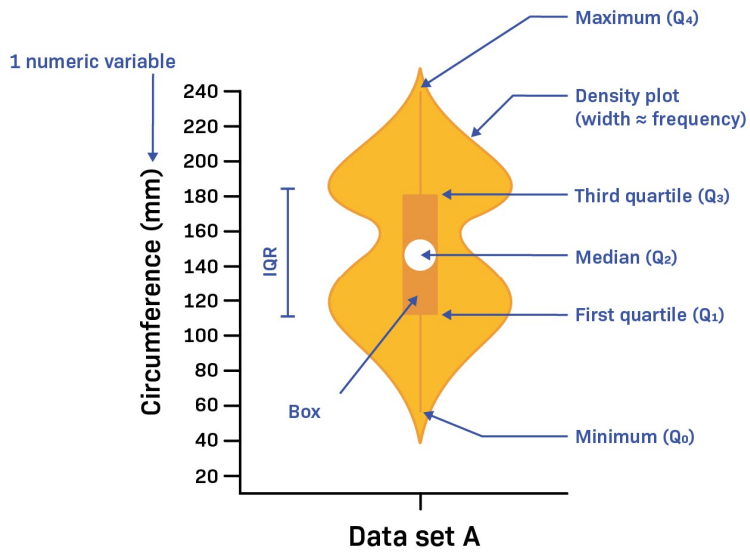
WARNING!! When you look at a box plot you might think that a larger part of the box represents more results than a smaller part of the box . This is incorrect! In each of the four parts of a box plot (approximately) the same number of observations is represented. The size of the parts of the box do hence not give an indication of the number of represented observations.

A larger part of the box, like in our example between 9 and 12, means that the observations here are more spread out. Between 12 and 14 the observations are hence less spread out. In each of the four parts of this box plot approximately 25% of the exam results are represented.

	Box plot only items		
	No ref	Ref	
No MERs	55.02	73.45	65.32
MERs	51.02	77.83	64.00
	52.95	75.38	64.69

Note: MERs = multiple external representations. Ref = refutational text.



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ORIGINAL RESEARCH article
Front. Educ., 29 August 2024
Sec. STEM Education
Volume 9 - 2024 | https://doi.org/10.3389/feduc.2024.1425663

How can eye-tracking data be used to understand cognitive processes when comparing data sets with box plots?

Martin Abt [1*] Timo Leuders [1] Katharina Loibl [2] Anselm R. Strohmaier [3]
Wim Van Dooren [4] Frank Reinhold [1]

Abt, M., Leuders, T., Loibl, K., Strohmaier, A., Van Dooren, W., Reinhold, F. (2024). How can Eye-Tracking Data be Used to Understand Cognitive Processes when Comparing Data Sets with Box Plots? *Frontiers in Education*, *9*. doi: 10.3389/feduc.2024.1425663

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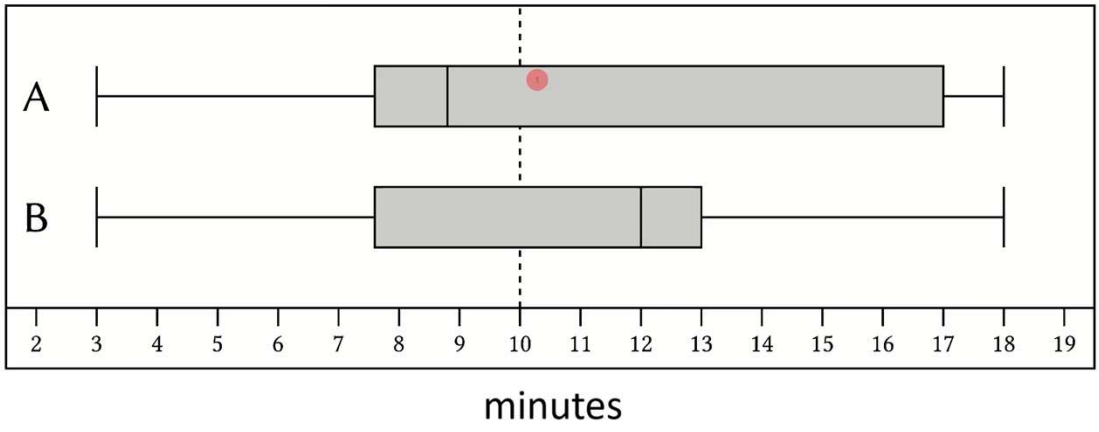
One school has a larger number of children with a walk to school longer than 10 minutes.

Decide which one.



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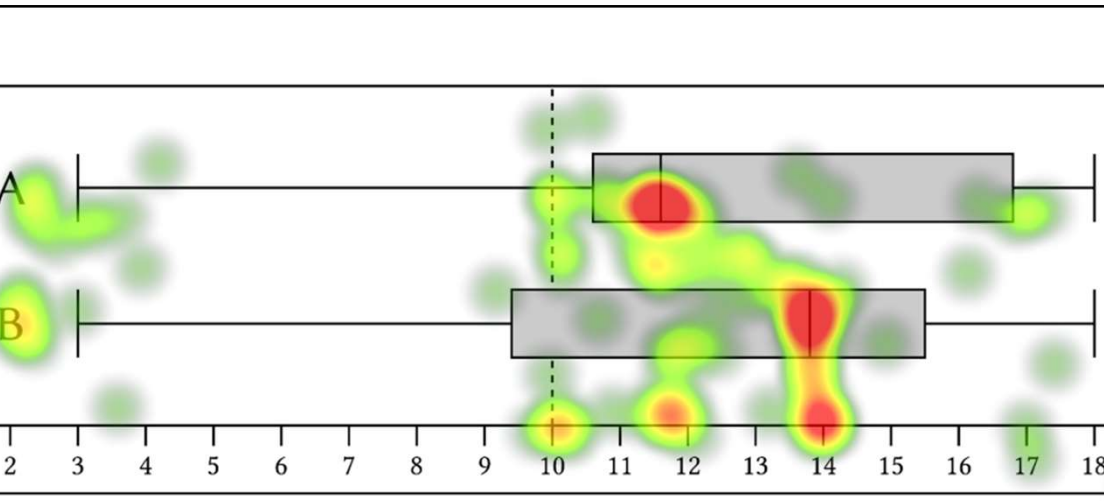

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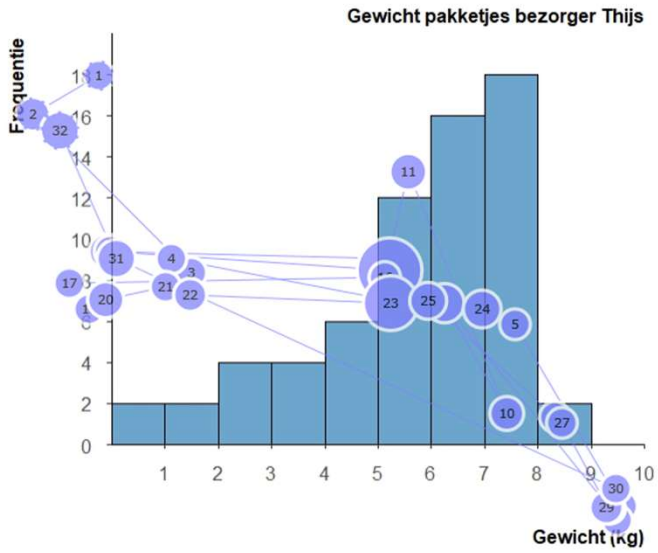

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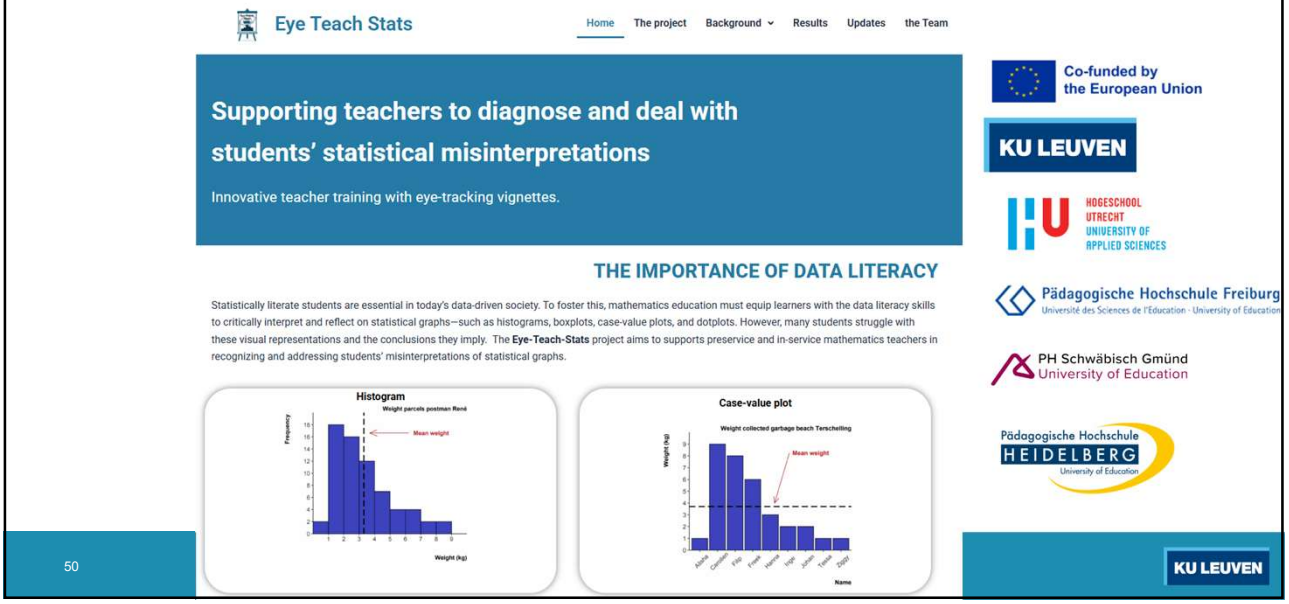
Eye Teach Stats https://eye-teach-stats.eu/



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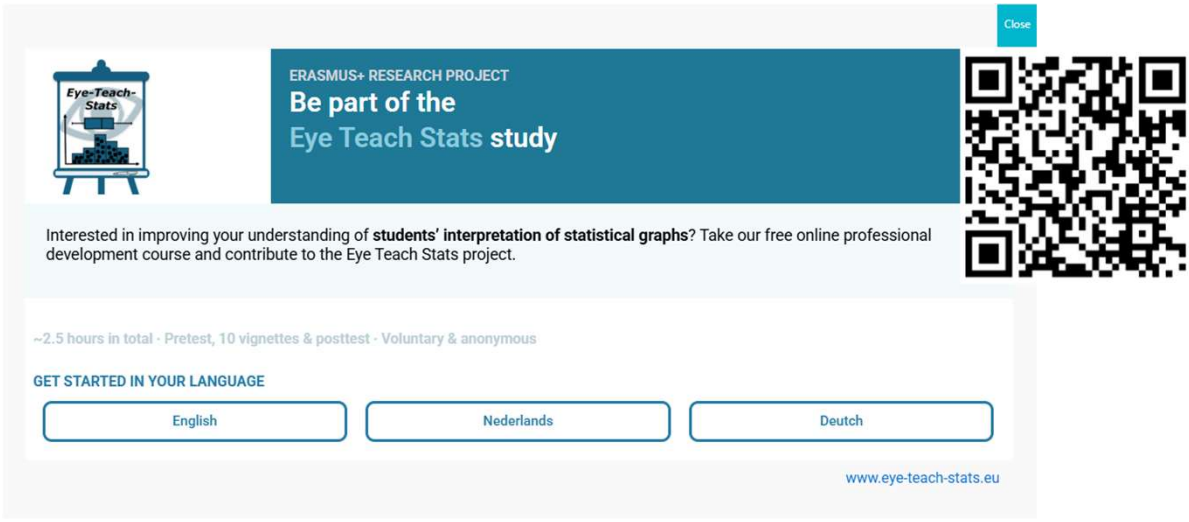
Eye Teach Stats https://eye-teach-stats.eu/



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Online omgeving

- https://www.soscisurvey.de/pretest_ckpck/?q=base

- Intake survey: +/- 20 minuten
  Geeft toegang tot de vignettes
- Online cursus met vignettes (kan je op je eigen tempo uitvoeren)
- Exit survey

→ Deel gerust met mede-leerkrachten en lerarenopleiders
→ Materiaal voor leerlingen volgt

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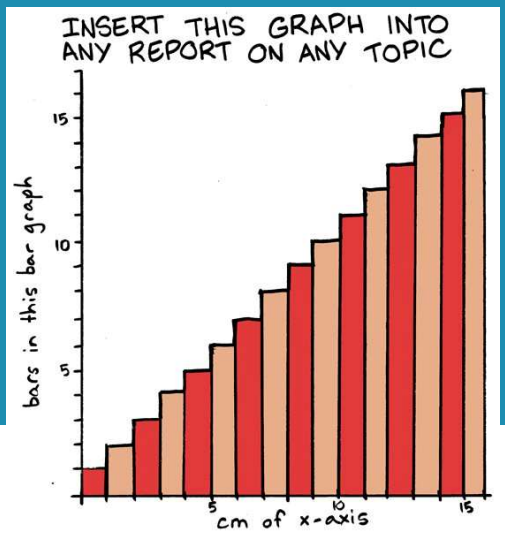
Dankjewel!



wim.vandooren@kuleuven.be

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